[2] Elata, C., Lehrer, J., and Kahanovitz, A., "Turbulent Shear Flow of Polymer Solutions," *Israel Journal of Technology*, Vol. 4, 1966, pp. 87–95.
[3] Virk, P. S., "The Toms Phenomenon—Turbulent Pipe Flow of Dilute Polymer Solutions," Ph.D. thesis, 1966, M.I.T., Cambridge, Mass.
[4] Van-Driest, E. R., "The Damping of Turbulent Flow by Long Chain Molecules," Scientific Report AFOSR 67-2369, 1967, Ocean System Operations, North American Rockwell Corp., Anaheim, Calif.
[5] Seyer, F. A. and Metzner, A. B., "Turbulence Phenomena in Drag Reducing Systems," presented at 60th Annual AIChE Meeting, New York, 1967.
[6] Poreh, M. and Paz, U., "Turbulent Heat Transfer to Dilute Polymer Solutions," *International Journal of Heat and Mass Transfer*, Vol. 11, pp. 805–818.
[7] Von-Kármán, T., "The Analogy Between Fluid Friction and Heat Transfer," *Transactions of the ASME*, Vol. 61, 1939, p. 705.
[8] Levich, V. G., *Physicochemical Hydrodynamics*, Prentice-Hall, Englewood Cliffs, N.J., 1962, pp. 139–157.
[9] Wells, C. S., "An Analysis of Uniform Injection of a Drag Reducing Fluid into a Turbulent Boundary Layer," *Viscous Drag Reduction*, edited by C. S. Wells, Plenum Press, New York, 1969, pp. 361–382.
[10] Friend, W. L. and Metzner, A. B., "Turbulent Heat Transfer inside Tubes and the Analogy among Heat Mass and Momentum Transfer," *AIChE Journal*, Vol. 4, 1958, pp. 393–402.
[11] Suraiya, T., "Mass Transfer to Dilute Polymer Solutions in Turbulent Pipe Flow," B.Sc. thesis, 1968, M.I.T., Cambridge, Mass.

Dynamic Response of a Cylindrical Shell in a Resistant Medium

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Introduction

The dynamic response of rigid-plastic circular cylindrical shell was first investigated by Hodge.[1] Amandsov[2] continued the work by including a resistant force proportional to the first power of the velocity. Venkatraman and Patel[3] later solved the problem for an orthotropic shell. However, all these analyses used simplifying assumptions; the first two investigators incorporated the square yield condition whereas Venkatraman and Patel used a hexagon yield condition for an orthotropic shell, but neglected the effect of the time-dependent interface coordinate between the two plastic regimes. The purpose of this Note is to study the effect of these simplifying assumptions upon the maximum deflection of the shell.

The problem considered herein is the dynamic response of a short rigid-plastic circular cylindrical shell subjected to an exponentially decaying pressure which is uniform along the length. The shell is initially at rest, clamped at both ends, and the material is assumed to behave according to an inscribed hexagonal curve to the exact tresca yield condition (Fig. 1). The effect of a resistant force, which is proportional to the first power of the velocity, is included.

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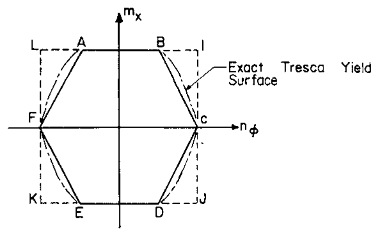

Fig. 1 Yield conditions.

Analysis

Consider a circular cylindrical shell of length $2L$, radius a, and thickness $2h$. The axial component of the length x originates at a clamped end. The shell is loaded by a uniform pressure p. The resistant force is assumed to be proportional to the first power of the velocity, where the proportionality constant is the product of the velocity of sound in the medium and the density of the medium.[2] An inward radial displacement is assumed positive.

For simplicity, the following nondimensional quantities are introduced

$$\alpha^2 = L^2/ah;\ m_x = M_x/\sigma_0 h^2;\ n_\phi = N_\phi/2\sigma_0 h;\ w = saW/2\sigma_0 h t_0^2$$
$$p = pa/2\sigma_0 h;\ \tau = t/t_0;\ \gamma = t_0/s;\ y = x/L \quad (1)$$

The quantities M_x and N_ϕ are the axial bending moment and the circumferential force per unit length, while σ_0, and t_0 represent a convenient reference stress and time, respectively, and s is the mass per unit area of the shell. The governing equation for the shell can be written as

$$m_x''/2\alpha^2 + n_\phi + Pe^{-\tau} - \ddot{w} - \gamma\dot{w} = 0 \quad (2)$$

where the primes and dots indicate differentiation with respect to y and τ, respectively.

The short shell is assumed to behave similarly to a clamped beam under a uniform load. The appropriate stress profile from the solid line in Fig. 1 is from point E to F and from F to A. Point E corresponds to $y = 0$ and point A corresponds to $y = 1$. Point F represents some intermediate coordinate $y = \eta$ (interface or inflection point) which will be determined. Due to symmetry, only one half the shell length need be considered. The boundary conditions for the short shell are

$$m(\tau,0) = -1;\ m(\tau,\eta^-) = m(\tau,\eta^+) = 0;\ m(\tau,1) = 1$$
$$m'(\tau,\eta^-) = m'(\tau,\eta^+);\ m'(\tau,1) = 0$$
$$w(\tau,0) = \dot{w}(\tau,0) = 0;\ \dot{w}(\tau,\eta^-) = \dot{w}(\tau,\eta^+);\ \dot{w}'(\tau,\eta^-) = \dot{w}'(\tau,\eta^+) \quad (3)$$

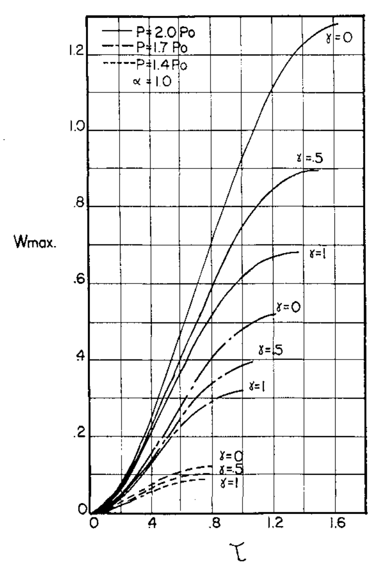

Fig. 2 Progress of maximum displacement.

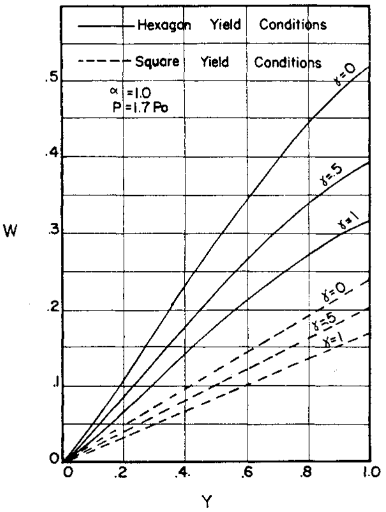

Fig. 3 Maximum displacements for hexagon and square yield conditions.

Solving for the velocity from the flow law yields: for $0 \leq y \leq \eta$

$$\dot{w} = \dot{w}_0 \sinh\alpha y \tag{4a}$$

for $\eta \leq y \leq 1$

$$\dot{w} = \dot{w}_0[\sinh\alpha y \cos\alpha(y - \eta) + \cosh\alpha y \sin\alpha(y - \eta)] \tag{4b}$$

where $\dot{w}_0$ is some function of τ. Substituting Eqs. (4) into Eq. (2) and solving subject to the boundary condition yields: for $0 \leq y \leq \eta$

$$m_x = 2(1 - Pe^{-\tau})\left[\frac{\sinh\alpha(\eta - y) + \sinh\alpha y}{\sinh\alpha\eta} - 1\right] - \frac{\sinh\alpha(\eta - y)}{\sinh\alpha\eta} + \alpha(\ddot{w}_0 + \gamma\dot{w}_0) \times \left[y \cosh\alpha y - \frac{\eta \cosh\alpha\eta \sinh\alpha y}{\sinh\alpha\eta}\right] \tag{5a}$$

for $\eta \leq y \leq 1$

$$m_x = 2(1 - Pe^{-\tau})\left[1 - \frac{\sin\alpha(1 - y) + \sin\alpha(y - \eta)}{\sin\alpha(1 - \eta)}\right] + \frac{\sin\alpha(y - \eta)}{\sin\alpha(1 - \eta)} + \alpha(\ddot{w}_0 + \gamma\dot{w}_0)\left[(y - 1)\sinh\alpha\eta \sin\alpha(y - \eta) - y \cosh\alpha\eta \cos\alpha(y - \eta) + \frac{\eta \cosh\alpha\eta \sin\alpha(1 - y)}{\sin\alpha(1 - \eta)} + \frac{\cosh\alpha\eta \sin\alpha(y - \eta)}{\tan\alpha(1 - \eta)}\right] \tag{5b}$$

Applying shear continuity at $y = \eta$ and the condition of zero shear at $y = 1$ to Eqs. (5) and solving the resultant equations simultaneously yields

$$2(1 - Pe^{-\tau})[\phi_2\phi_4 - \phi_1\phi_5] + \phi_3\phi_5 - \phi_2\phi_6 = 0 \tag{6a}$$

$$\ddot{w}_0 + \gamma\dot{w}_0 = [\phi_3 - 2(1 - Pe^{-\tau})\phi_1]/\phi_2 \tag{6b}$$

where

$$\phi_1 = (\cosh\alpha\eta - 1)/\sinh\alpha\eta + [1 - \cos\alpha(1 - \eta)]/\sin\alpha(1 - \eta) \tag{7a}$$

$$\phi_2 = 2\cosh\alpha\eta - \frac{\alpha\eta}{\sinh\alpha\eta} + \alpha(1 - \eta) \times \left[\sinh\alpha\eta - \frac{\cosh\alpha\eta}{\tan\alpha(1 - \eta)}\right] \tag{7b}$$

$$\phi_3 = 1/\sin\alpha(1 - \eta) - 1/\sinh\alpha\eta \tag{7c}$$

$$\phi_4 = [1 - \cos\alpha(1 - \eta)]/\sin\alpha(1 - \eta) \tag{7d}$$

$$\phi_5 = \sinh\alpha\eta \sin\alpha(1 - \eta) - \cosh\alpha\eta \cos\alpha(1 - \eta) + \alpha(1 - \eta)\frac{\cosh\alpha\eta}{\sin\alpha(1 - \eta)} \tag{7e}$$

$$\phi_6 = -\cot\alpha(1 - \eta) \tag{7f}$$

Eq. (6b) can be solved numerically for a given α and P by starting at the instant of load ($\tau = 0$) and incrementing τ until the shell comes to rest. For each increment of τ, however, the root of Eq. (6a) must first be obtained to find the numerical value of the nonhomogeneous part of Eq. (6b). The displacements can then be obtained for any value of τ by numerically integrating Eq. (6b).

Numerical Example and Comparison

For purposes of discussion and comparison, consider a shell whose $\alpha = 1$. Figure 2 shows the maximum displacements at ($y = 1$) for different loads and resistant forces. The influence of the resistant force increases as the pressure increases. For pressures slightly larger than the static collapse pressure, the resistant force can be neglected. For example, the decrease in the final displacement at the center of the shell is less than 1% due to the maximum resistant force and a load 10% above the static collapse pressure. For a pressure twice the static collapse load, however, a 46% difference exists between the final maximum displacements for a maximum resistant force and no resistant force.

Figure 3 shows a comparison of solutions obtained by the authors (hexagonal yield condition) and Hodge[1] and Amandsov[2] (square yield condition). This comparison shows a difference ranging between 50 and 55%, depending on the resistant force. It should be noted that the square yield condition underestimates the displacements compared to the hexagonal yield curve. In fact, the use of the hexagonal yield curve slightly overestimates the actual displacements and can therefore be considered as conservative.

A further comparison can be made between the authors' work (moving inflection point) and the method advanced by Venkatraman and Patel[3] (stationary inflection point). They reasoned that the inflection point moved less than 2% for a load 10% above the static collapse pressure, and could therefore be considered as stationary. However, the authors found that the percentage difference in the final displacements between the two methods were not sensitive to the magnitude of the load as suggested or to the resistant force. Rather, the errors introduced by assuming a static inflection point are quite sensitive to the shell parameter α. For example, errors of approximately 13, 16.5, and 22.5% are experienced for shell parameters of 0.5, 1.0, and 1.5, respectively, using any multiple of static collapse load and comparable resistant forces.

References

[1] Hodge, P. G., Jr., "Impact Pressure Loading of Rigid-Plastic Cylindrical Shells," *Journal of Mechanics and Physics of Solids*, Vol. 3, 1955, pp. 176–188.

[2] Amandsov, A. A., "Motion of a Rigid-Plastic Circular Cylindrical Shell in a Resisting Medium," *Proceedings of the 4th All-Union Conference on Shells and Plates*, Oct. 24–31, 1962, pp. 173–179.

[3] Venkatraman, B. and Patel, S. A., "Dynamic Response of Orthotropic Plastic Cylindrical Shells Under Radial Pressure," *Journal of the Franklin Institute*, Vol. 282, No. 3, Sept. 1966, pp. 171–178.